Seamless Phantom Sensation Moving across a Wide Range of Body

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Abstract— **This paper reports experimental research aimed to provide seamlessly moving illusory tactile sensations across a large area of the body using a few vibration actuators. The human vibrotactile sensitivity differences among the body sites are calibrated using empirically-measured psychophysical magnitude functions. We present a new phantom sensation rendering method that uses the Gaussian function emphasizing the spatial continuity of perceived movement and the temporal consistency of perceived intensity. We demonstrate that our rendering method outperforms previous methods for phantom sensations via a perceptual experiment. Our method is tailored to eliciting the perception of illusory tactile sensation moving in a long distance, from the thigh to the upper back.**

I. Introduction

Vibrotactile feedback is one of the most efficient ways to re-create haptic experiences of the real world in virtual environments [1], [2]. A direct approach to afford immersive haptic experiences is increasing the spatial resolution of a vibrotactile display. However, it is practically impossible to increase the resolution to the level of human tactile acuity [3]. To overcome this limitation, perceptual illusions are frequently utilized because of their effectiveness in rendering and simplicity in implementation. One of the most popular tactile illusions is phantom sensation, which refers to an illusory sensation that is felt between multiple vibrotactile stimulation points in close proximity to the skin. This work was motivated by our on-going research that uses a chair as a daily tactile interface (Fig. 1) and a question whether we can render seamless phantom sensations on such a platform.

A. Related Work

There have been plenty of studies about tactile phantom sensations after the initial studies by Békésy and Alles [4], [5]. Phantom sensations can be static or moving [6]. The feasibility of continuously moving phantom sensation was verified with vibration actuators directly stimulating the skin [7] or a physical medium transmitting vibration, e.g., a mobile device [8], [9]. Researchers also sought for rendering methods that provide better illusory moving sensations by changing the profiles of vibration commands [6], [10]–[12]. For that purpose, the linear and logarithmic modulation profiles have been most widely used, and it is known that they result in phantom sensations with different perceptual qualities [5], [8]. There was also an attempt to quantify the perceptual attributes of phantom sensations flowing through a mobile device when they were rendered using general polynomials [12]. Furthermore, multiple actuators more than two can be used to extend a range of 1D phantom sensations [10], [13] or to expand a dimension to render 2D phantom sensations moving on a plane [6], [9], [14]–[16].

In spite of such intensive research endeavors, the vast majority of the previous studies tested illusory sensations on a relatively small area on the body. Examples include the forearm [10], [13], the hand grasping a handheld device [6], [9], [15], part of the back [14], and a finger [16], where the stimulation points are in close proximity. Hence, to take full advantage of the perceptual illusion, it is crucial to design and verify methods for rendering phantom sensations moving across a wide range of the body using a few distant actuators.

B. Research Overview

This study was motivated by our on-going research project in which we seek effective methods in providing benefits by tactile feedback to drivers in vehicles. In this situation, a chair is a natural choice as a tactile display for drivers.

The research problem we address in this paper is whether phantom sensations that move in a long longitudinal distance can be rendered on the dorsal side of the body (Fig. 1), assuming a user sitting in a chair. Our first task was to find a way to compensate for the differences in vibrotactile sensitivity at such distant body sites. We estimated four psychophysical magnitude functions for four dorsal body parts by conducting a psychophysical experiment (Exp. I) that used the absolute magnitude estimation procedure and then fitting the obtained data to Stevens' power law. These magnitude functions enabled us to design a new rendering rule for phantom sensations that varies the *perceived* intensities of vibrations using Gaussian profiles.

The performance our rendering method was verified by another perceptual experiment. Participants assessed the quality of moving phantom sensations with respect to the spatial continuity of movement and the temporal consistency of intensity. For comparison, two conventional methods, one using a linear profile and the other using a logarithmic profile, were included. While these two methods specify the vibration intensity using a physical unit, our Gaussian rendering method uses a profile representing perceived intensity. As such, we included another profile that varies perceived intensity linearly [7]. Experimental results supported the adequacy of our new rendering rule to creating long phantom sensations moving in the longitudinal direction.

All perceptual experiments reported in this paper were approved by the Institutional Review Board at the authors' institution (PIRB-2018-E093).

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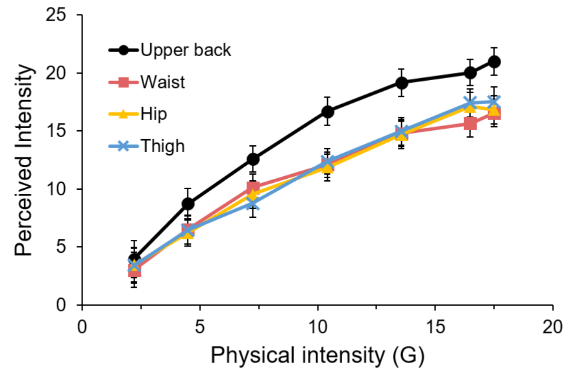
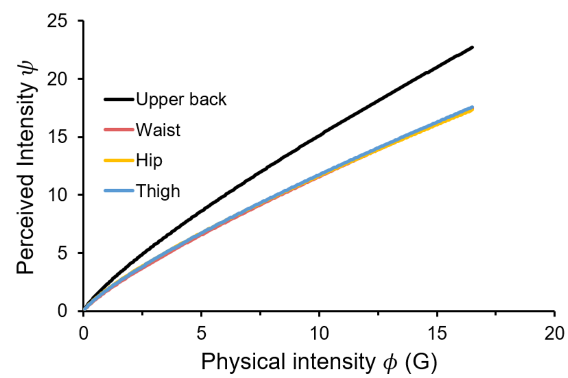
Fig. 1. Seat-type vibrotactile display and actuator configurations.



(a) Magnitude estimates for each contact site.



(b) Results of regression to Stevens' power law.

Fig. 2. Results of Exp. I. Error bars represent standard errors.

II. Exp. I. Psychophysical Magnitude Functions

Our research goal was to find an effective method to elicit seamless phantom sensations perceived to be moving across a large distance on the body from the thigh to the upper back. Since vibrotactile magnitude perception depends on the body site under stimulation [2], it can be advantageous to obtain the psychophysical magnitude functions that account for the mappings from the physical intensity of vibrotactile stimulation to the perceived intensity for each contact site and then consider the functions in the design of algorithms rendering phantom sensations. This was the aim of Exp. I reported in this section.

A. Methods

1) Participants: Fifteen right-handed people (13 males and 2 females; 21–27 years old with an average 24.3) participated in Exp. I. None of them reported any sensorimotor disorder. The experiment took about 40 min, and the participants were paid about USD 7 after the experiment.

2) Apparatus: We designed a seat-type vibrotactile display shown in Fig. 1. Four voice-coil actuators (DAYTON, PUCKTM mini bass shaker, TT25-8) were attached on the right side of a reclining chair. The actuators were 23 cm apart vertically from each other and stimulated the human's thigh, hip, waist, and upper back. Vibration from each actuator was well localized to the respective contact area owing to the use of small pieces of surrounding sponge in the chair.

3) Stimuli: This experiment used 28 stimuli composed of 7 amplitude levels (2.20, 4.48, 7.25, 10.41, 13.56, 16.48, and 17.50 G) of 100-Hz sinusoidal vibration and 4 body sites (thigh, hip, waist, and upper back on the right). We chose 100-Hz vibrations since they feel pleasant on the four body areas. Each stimulus was 1-s long.

4) Procedure: We conducted a training session before the main session. Participants were instructed to sit in the chair comfortably while maintaining good contact with the chair, not twisting their legs, bending over, or changing the posture during the experiment. Then participants experienced all of the stimuli once in random order for familiarization.

The main experiment followed the general procedure of absolute magnitude estimation with free modulus [17]. On each trial, a vibration was presented to a participant. Then the participant typed in a positive number that was deemed to best represent the perceived intensity of the stimulus. Standard instructions taken from [18], which stresses the availability of infinitely many numbers between any real numbers, were given to participants before the experiment.

The main experiment consisted of six sessions for each contact site. In each session, all of the 28 stimuli were presented in random order. We gave participants one minute for a break between sessions. The results of the first two sessions were discarded and not used for data analysis. During the experiment, participants were presented with 100-Hz sound through headphones to block auditory cues from vibration propagating in the body. The order of contact sites to test was also randomly determined for each participant.

5) Data Processing: In absolute magnitude estimation experiments, data standardization is generally required to reduce deviations across participants. We used the mean deviation standardization method in [19].

Then, the data collected at each contact site were fed to a fitting procedure using Stevens' power law [20]. Stevens' power law is one of the most established empirical laws in cognitive psychology, and it states that

$$\psi = k\phi^{\alpha},$$

where ψ is the perceived intensity, ϕ is the stimulus intensity, α is the power exponent that depends on the sensory modality and stimulus conditions, and k is an arbitrary constant.

B. Results

Fig. 2a shows the results of magnitude estimation for the four body parts. At every stimulus intensity level, the perceived intensities on the upper back had the highest values among the four contact sites. The other three body sites resulted in similar perceived intensities. Two-way repeated-measures ANOVA showed that body site had a significant effect on perceived intensity ($F(3, 42) = 30.24, p < 0.0001$).

Stevens' power functions derived from the raw data in Fig. 2a are shown in Fig. 2b for the four body sites of thigh, hip, waist, and upper back. Note that we used the data at only the first six physical intensity levels. We noticed during the experiment that the data at the greatest physical intensity (17.50 G) were unreliable since it reached the actuator limit.

The equations of the four psychophysical magnitude functions were as follows:

$$\begin{aligned} \psi_{thigh} &= 1.843\, \phi^{0.8044} \quad (R^2 = 0.9977) \\ \psi_{hip} &= 1.885\, \phi^{0.7910} \quad (R^2 = 0.9980) \\ \psi_{waist} &= 1.763\, \phi^{0.8177} \quad (R^2 = 0.9770) \\ \psi_{upperback} &= 2.333\, \phi^{0.8117} \quad (R^2 = 0.9747) \end{aligned}$$

All the coefficients of determination (R^2) were higher than 0.97, indicating very good fit. The values of power exponent were all less than 1, exhibiting saturation behaviors.

III. Exp. II: Moving Phantom Sensations

In this section, we present a new rendering method for moving phantom sensations designed for a long traveling distance across the longitudinal direction of the body. Our method was empirically compared with conventional methods in Exp. II, in terms of several measures regarding the quality of phantom sensations.

A. Rendering Methods

Previous studies on moving phantom sensations generally use three methods. Two of them are to modulate the physical intensity of vibration linearly (LINEAR) and logarithmically (LOG), originated from the work by Alles [5]. To explain the detail, consider multiple actuators that are placed on a line. We want a moving phantom sensation to pass the actuators through sequentially at a constant speed, and it should take the same time T to move from one actuator to the next actuator. The physical vibration amplitude A_i of the i-th actuator at time t is computed by the following equations:

$$A_i^{\mathrm{LINEAR}} = \begin{cases} A_{max}\left(1 - \dfrac{|t - Ti|}{T}\right) & \text{if } |t - Ti| < T \\ 0 & \text{otherwise} \end{cases},$$

$$A_i^{\mathrm{LOG}} = \begin{cases} A_{max}\log_2\left(2 - \dfrac{|t - Ti|}{T}\right) & \text{if } |t - Ti| < T \\ 0 & \text{otherwise} \end{cases},$$

where A_i^{LINEAR} is for LINEAR, A_i^{LOG} is for LOG, and A_{max} is the maximum physical intensity of a stimulus.

The other method is to modulate the perceived intensity of vibration linearly (PI-LINEAR), as alluded in [10]. The perceived intensity ψ_i of the vibration generated by the i-th actuator is determined by

$$\psi_i^{\mathrm{PI-LINEAR}} = \begin{cases} \psi_{max}\left(1 - \dfrac{|t - Ti|}{T}\right) & \text{if } |t - Ti| < T \\ 0 & \text{otherwise} \end{cases},$$

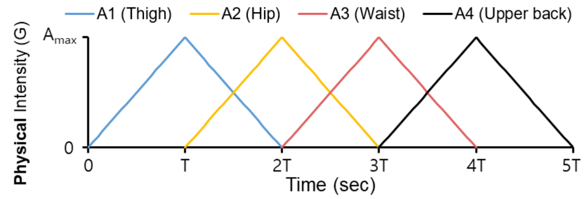
where ψ_{max} is the maximum perceived intensity.

While testing the three methods with our vibrotactile chair, we encountered a few problems. The most notable was that the movement of phantom sensation is not smooth, especially when it passes on the real stimulation points. Sometimes a moving sensation even appeared or disappeared along the way.

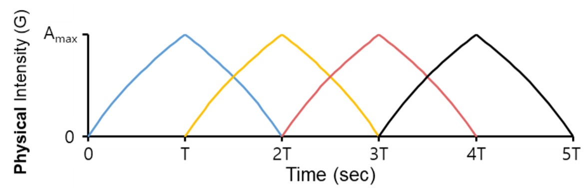
To overcome these issues, we devised a new rendering method of moving phantom sensation that is suitable for a tactile display consisting of three or more actuators spanning a long range on the body. This method, named PI-GAUSSIAN, modulates the perceived intensity using the Gaussian function, as follows:

$$\psi_i^{\mathrm{PI-GAUSSIAN}} = \psi_{max} \exp\left\{-\frac{1}{2\sigma^2}\left(\frac{t - Ti}{T}\right)^2\right\},$$

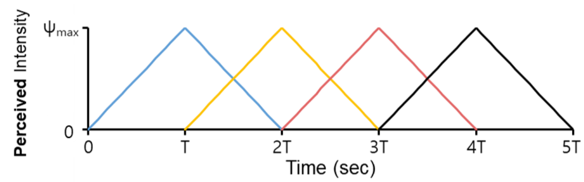
where σ is the standard deviation of Gaussian function. All of the actuators are involved in producing an illusory sensation



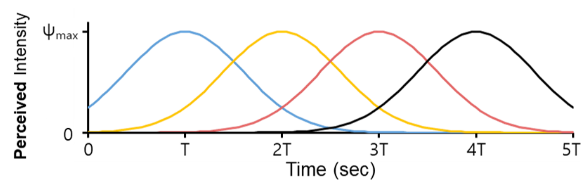
(a) LINEAR.



(b) LOG.



(c) PI-LINEAR.



(d) PI-GAUSSIAN.

Fig. 3. Four rendering methods for phantom sensations.

in PI-GAUSSIAN although their intensities may be weak, while only two actuators produce stimulation in the other methods. In this way, we intend to reduce the discontinuity that is caused by the sudden appearances and disappearances of vibrations at the real actuators. Besides, because the PI-GAUSSIAN function is differentiable everywhere, the change of the intensity is always smooth while those of the other methods are not.

We used four actuators ($i = 1, ..., 4$) that were in direct contact with the thigh, hip, waist, and upper back, respectively. A moving sensation can be felt while time t increases from T to $4T$. The increasing interval between 0 and T and the decreasing interval between $4T$ and $5T$ are for continuous start and end of the stimulus, preventing sudden appearance or disappearance.

In our study, we set A_{max} to 16.48 G, which was the maximum physical intensity used to derive the psychophysical magnitude functions in Exp. I. We set ψ_{max} to 17.29, and it was the minimum perceived intensity among the four body locations at the maximum physical intensity (16.48 G). We also chose to use $\sigma = 0.6$ after pilot tests since it elicited the most plausible illusion. The graphs of intensity over time t are shown in Fig. 3 for the four methods.

B. Measures for Evaluation

The ideal situation we assume is that a real actuator playing vibration with a constant intensity is moved on the skin at a constant velocity while maintaining contact with a constant pressure. In such a case, the movement of stimulation point should be continuous (continuity), and the stimulation should feel to have the same strength (consistency). Thus, we use the continuity of illusory movement and the consistency of perceived intensity as two measures for evaluating the quality of moving phantom sensation.

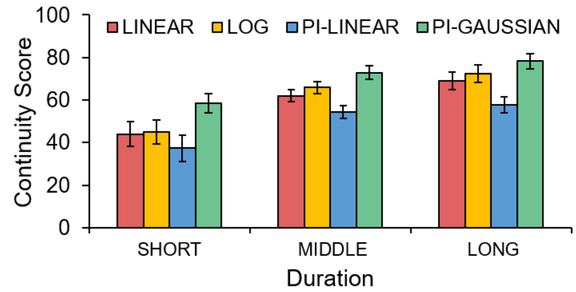
Similar measures were used in previous studies. For example, the continuity score of moving sensation was measured in [10] and [13]. The position and intensity profiles of phantom sensation were estimated in [11] and [12]. These studies also analyzed whether the intensity was maintained consistently by looking at its variance.

Hence, we define a desirable moving phantom sensation to be one that feels *moving continuously* with *consistent intensity*. In the main experiment, we asked two questions: "How continuously is the stimulus moving?" (continuity) and "How consistent is the variation of vibration intensity?" (consistency). The consistency score was reverse-coded.

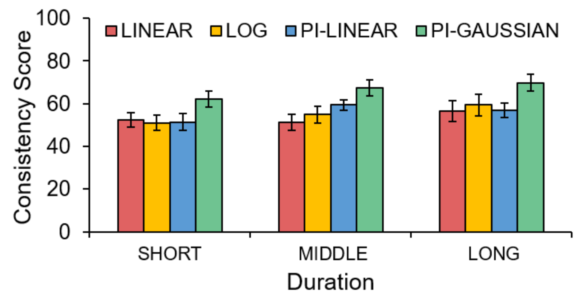
C. Methods

For brevity, the methods used commonly in Exp. I are not repeated in this section.

1) Participants: Twenty right-handed people (17 males and 3 females; 18–28 years old with an average 23.9) without any sensorimotor disorder participated in Exp. II. None of the participants also participated in Exp. I. The experiment took about 45 minutes, and participants were paid about USD 8 after the experiment.



(a) Continuity score.



(b) Consistency score.

Fig. 4. Results of Exp. II. Error bars represent standard errors.

2) Stimuli: We presented moving phantom sensations generated by two independent factors, rendering method and stimulus duration. The experiment had 12 conditions of stimuli with four rendering methods (LINEAR, LOG, PI-LINEAR, PI-GAUSSIAN) and three durations ($T = 0.15, 0.3, 0.4$ s). The whole duration of the sensations was 0.75 (SHORT), 1.5 (MIDDLE), and 2.0 s (LONG).

3) Procedure: Participants finished a training session first. Phantom sensations are illusory, and it takes sufficient exposure to the stimuli for people to become familiar with the illusory sensations and make robust perceptual judgments. In the training session, participants were asked to feel each stimulus at least four times. The training session lasted until participants felt that they had experienced the stimuli sufficiently to evaluate them. After the training session, we asked participants to explain their subjective feelings about the stimuli verbally.

In the main experiment, each condition of the 12 stimuli was presented to participants in random order. Participants rated both the continuity and the consistency scores from 0 to 100 after perceiving each stimulus. We provided scales from 0 to 100 in units of 10 and the descriptions of the extremes and the midpoint to help participants respond. Participants were able to replay a stimulus if they wanted. The main experiment consisted of five sessions, and the 12 stimuli were repeated once in each session. Participants were given one minute of a break between sessions. We presented 100 Hz sound through headphones to block auditory cues during the whole time of the experiment including the training session.

4) Data Analysis: In the analysis step, we excluded the results of the first session and used those of the later four

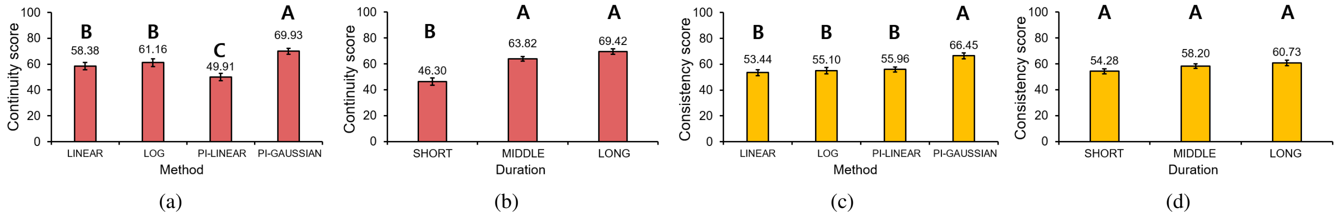


Fig. 5. Average scores of continuity and consistency scores broken down by rendering method and stimulus duration. Letters represent the results of SNK post-hoc tests. The conditions grouped with the same letters did not show statistically significant differences in their means.

sessions. We then averaged the continuity and consistency scores of each condition across participants.

D. Results

The mean continuity and consistency scores are shown in Fig. 4. To figure out which factors had significant effects on the scores, we applied two-way repeated-measures ANOVA to the continuity and the consistency scores, respectively. We observed that both rendering method ($F(3, 57) = 27.06, p < 0.0001$) and stimulus duration ($F(2, 38) = 10.06, p = 0.0003$) had significant effects on the continuity score. However, the interaction between them did not ($F(6, 114) = 0.2731, p = 0.2731$). In terms of the consistency score, rendering method was a significant factor ($F(3, 57) = 7.58, p = 0.0002$) while stimulus duration and their interaction did not have a significant effect ($F(2, 38) = 2.11, p = 0.1346$; $F(6, 114) = 2.01, p = 0.0698$). For further comparisons, we ran Student-Newman-Keuls (SNK) post-hoc analysis on the significant main effects. The results of this post-hoc analysis are shown in Fig. 5.

The four rendering methods were divided into three groups as to continuity. The lowest continuity score was observed with PI-LINEAR, which was 49.91. The continuity scores of LINEAR and LOG were 58.38 and 61.16, and they were not significantly different from each other. The continuity score of PI-GAUSSIAN was the highest with 69.93.

As for stimulus duration, the continuity score of SHORT was the lowest (46.30). The continuity scores of MIDDLE and LONG were 63.82 and 69.42, and they were not significantly different from each other. Also, the continuity score increased as the duration increased.

PI-GAUSSIAN had the highest consistency score (66.45). The consistency scores of LINEAR (53.44), LOG (55.10), and PI-LINEAR (55.96) were not significantly different.

E. Discussion

In this experiment, we evaluated the quality of moving phantom sensations rendered by three previous methods and one new method of PI-GAUSSIAN. We were interested in whether the two independent factors, rendering method and stimulus duration, had significant effects on the perceived continuity and consistency of moving phantom sensation.

1) Effects of Rendering Method on Continuity: Our definition for the continuity score of phantom sensation emphasized the spatial continuity of illusory sensation movement. The continuity score was improved in order of PI-LINEAR $<$ LINEAR $<$ LOG $<$ PI-GAUSSIAN. PI-GAUSSIAN had the best continuity score with statistical significance.

We can compare the profiles of *perceived intensity* among the four rendering methods. This is illustrated in Fig. 6a, where the actuator makes the greatest vibration at time 2T. It can be seen that the sharpness with which the perceived intensity changes at the peak is ordered as PI-GAUSSIAN $<$ LOG $<$ LINEAR $<$ PI-LINEAR. This is the exactly revered order of the continuity score. It suggests that the smoothness of perceived intensity changes, especially at the peaks, is a key to achieving the phantom sensations that are perceived to move continuously. In this regard, the best result of PI-GAUSSIAN can be attributed to the use of the Gaussian function that is continuous and differentiable everywhere.

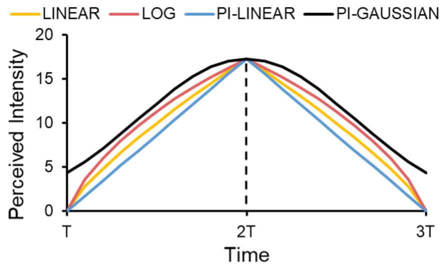
When one actuator renders the vibration of the maximum intensity, the other actuators cease to make vibration in the three previous rendering methods (Fig. 3). In contrast, PI-GAUSSIAN still supplies weak vibrations using the other actuators. These weak vibrations may prevent a sudden appearance of vibration when the actuation starts. We received a subjective report of the sudden vibration appearance problem for the three other methods.

Our findings are applicable to phantom sensations moving over a long distance using three or more actuators. They are not for the two actuator case, where the intensity of one actuator either only increases or decreases.

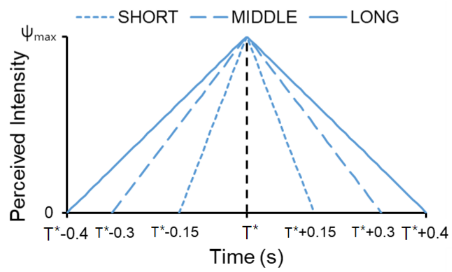
2) Effects of Stimulus Duration on Continuity: The continuity score increased with stimulus duration, although the scores of MIDDLE and LONG were not significantly different. The shorter the stimulation duration is, the sharper the changes in perceived intensity are (Fig. 6b). So using a stimulus of longer duration can improve the smoothness aspect of phantom sensation.

3) Effects of Rendering Method on Consistency: Vibrotactile sensitivity depends on the body site. Stimulating different body sites with identical physical intensity can cause the perception of vibrations with different strength. This may work adversely to our measure of consistency, which pertains to the uniformity of perceived intensity over time. This general fact may be relevant to our results between rendering method and consistency.

PI-GAUSSIAN showed the highest consistency score that was statistically different from the scores of the other three methods. The consistency scores were not statistically different among PI-LINEAR, LOG, and LINEAR, but the score of PI-LINEAR was higher than the others. Thus, the two



(a) Four rendering methods.



(b) Three stimulus durations (PI-LINEAR)

Fig. 6. Perceived intensity profiles computed using the psychophysical magnitude function on the hip.

methods, PI-GAUSSIAN and PI-LINEAR, which provide the vibrations that are calibrated for perceptual strength across the body sites, seem to outperform the others.

According to subjective reports, several participants reported that they had felt strong hitting sensations on the upper back when phantom sensations were rendered using LINEAR and LOG, but not for PI-LINEAR or PI-GAUSSIAN. This can be due to the result of Exp. I that the perceived intensity of 100 Hz vibration is the largest on the upper back out of the other three body sites. In LINEAR and LOG, this difference in sensitivity is not compensated for. With the two methods, participants could have felt vibrations of inconsistent intensity especially on the upper back, and it could have lowered the consistency scores of the two methods.

IV. Conclusions

The work reported in this paper aimed to design a rendering method for dynamic phantom sensations that deliver realistic illusory tactile sensations moving across a wide range of the body, from the thigh to the upper back, with a few actuators. In order to compensate for the sensitivity differences across the body, we measured four psychophysical magnitude functions of vibrotactile stimuli at the four body sites of thigh, waist, hip, and upper back in Exp. I. Then we designed a new phantom sensation rendering method that uses the Gaussian function in the profile of perceived intensity to support smoother transitions at even actuator locations. Our method was evaluated in Exp. II along with three other previous rendering methods. Results indicated that our method provides illusory motion of the best quality in terms of the spatial continuity of movement and the temporal consistency of intensity. Our findings can contribute to the appropriate design of tactile displays and rendering algorithms stimulating the body in a long distance.

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