# Perceptual Space of Regular Homogeneous Haptic Textures Rendered Using Electrovibration

Sunung Mun, Hojin Lee, and Seungmoon Choi

***Abstract*— This paper is concerned with the perceptual structure of homogeneous and deterministic haptic textures rendered by an electrovibration display. To represent textures, we modeled 32 regular tessellations of polygons by changing the polygon, density, edge width, pixel intensity, and image reversal. We conducted a perceptual experiment in order to estimate the pairwise dissimilarities between the 32 textures using the cluster sorting procedure. We then applied multi-dimensional scaling to the data and obtained a perceptual space that accounted for the structural effects of the five design variables on texture perception. The subjective impressions of the textures were also rated against eight adjective pairs. We projected the rating results into the perceptual space in order to find adequate perceptual dimensions describing the texture perception. Our results can contribute to designing perceptually distinctive textures elicited by electrovibration with an appropriate understanding of their subjective qualities.**

## I. Introduction

Electrovibration (electroadhesive) displays are tailored to rendering planar spatial patterns using relatively simple additional hardware. Despite their high potential for easy integration into the prevalent touchscreen devices, electrovibration displays can generate only lateral resistive force, but not lateral attractive or normal force. This limitation poses urgent needs for a comprehensive and in-depth understanding of the perceptual characteristics of the tactile stimuli provided by an electrovibration display. In this context, we address the perceptual space of electrovibration textures in this paper.

### A. Related Work: Electrovibration

Bau et al. demonstrated that electrovibration can be generated and used for user interaction using a transparent film mounted on a touchscreen [1]. Since then, electrovibration displays have received intensive attention from the haptics community. They are particularly appropriate for eliciting sensations of surface textures, and several methods have been proposed to this end. Meyer et al. used spatial spectrograms to represent and reconstruct textures [2] and designed a procedural texturing procedure using stochastic models [3]. Our group adopted the data-driven concept that captures texture models from real textures and designed a texture rendering method for an electrovibration display by identifying and using the inverse dynamics of the display [4]. Possibility for inducing the perception of 3D features using electrovibration has also been tested. Kim et al. rendered 3D features using a texture-like gradient-based technique [5]. We applied a similar gradient-based method to present 3D height [6], [7].

The perceptual characteristics of electrovibration have also been a good subject of research. Kang et al. found a way to decrease the input voltage amplitude while providing an electrovibration of the same perceptual strength [8]. Vardar et al. published a series of perceptual studies regarding electrovibration for the effects of waveform [9], roughness [10], and masking [11].

### B. Related Work: Perceptual Space

*Perceptual space* is a $n$-dimensional mathematical space that visualizes the relationships between the percepts of stimuli. It is a powerful tool for perception research supporting similarity and difference judgments while enabling further processing such as classification and naming [12]. Careful observation of the distribution of stimuli in a perceptual space often leads to revealing hidden perceptual structures. Since Hollins' seminal article on the perceptual dimensions of haptic textures [13], many studies have used perceptual space for various haptic stimuli, such as vibrations [14], [15], [16], [17] and textures [18], [19]; see [17] for review.

Research regarding perceptual space is often strengthened by additional efforts in which the subjective qualities of stimuli are compared against adjectives. However, in most languages, the adjectives that describe the perceptual properties of haptic stimuli are not as well developed as those for visual or aural stimuli. Thus, there has been a decent body of research that aims to associate existing words in some languages with tactile sensations [20], [21]. Such adjectives are rated for a set of stimuli, and results are mapped into a perceptual space of the stimuli. Some adjectives there may critically contribute to describing the perceptual structure of the stimuli, playing the role of *perceptual dimensions* [13], [15], [16], [19]. Perceptual space and adjective rating can be an effective pair of tools for looking into perception.

### C. Research Overview

Here we consider homogeneous and deterministic surface textures, and we express them using regular tessellations of two-dimensional (2D) polygons. A regular tessellation is a standard pattern made by repeating an edge-to-edge tile shaped in a regular polygon to create a uniform mosaic image [22]. We address a perceptual space of such regular textures when they are rendered via electrovibration, as an effort to obtain and accumulate knowledge on the perceptual characteristics of electrovibration stimuli. Varying five variables, we designed 32 regular textures and estimated their pairwise

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Authors are with the Haptics and Virtual Reality Laboratory, Department of Computer Science and Engineering, POSTECH, Republic of Korea.
E-mail: {mtablo, hojini33, choism}@postech.ac.kr

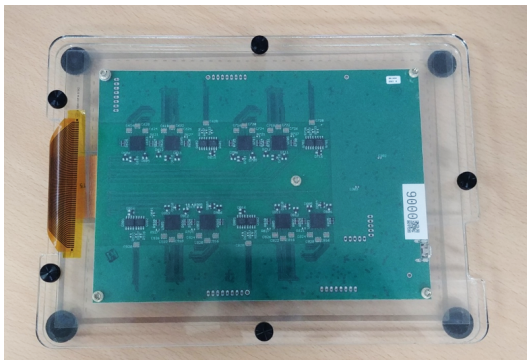


Fig. 1. TanvasTouch DK V1.0. A user touches and scans the top glass to feel electrovibration. This device is controlled by a separate tablet running an experiment program.

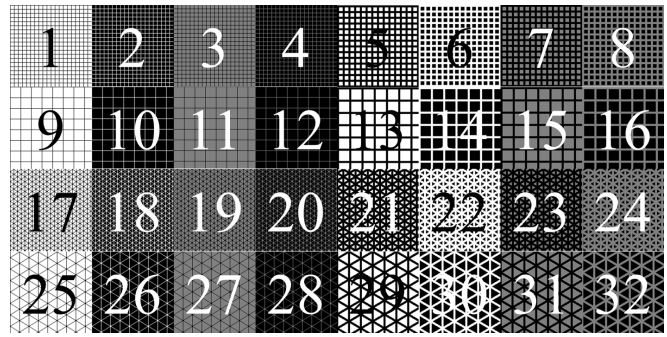



Fig. 2. Texture images generated by regular tessellation.

dissimilarities by a perceptual experiment using the cluster sorting procedure [23]. Applying multi-dimensional scaling (MDS) to the dissimilarity data, we obtained a perceptual space and then observed the relative significance of the five design variables for rendering distinctive textures. The subjective impressions of the textures were also rated against eight adjective pairs. We project the results into the perceptual space in order to find adequate perceptual dimensions describing the texture perception elicited by electrovibration.

## II. Texture Representation

### A. Apparatus

For an electrovibration display, we used a TanvasTouch DK (Development Kit) 1.0. It consisted of an electrovibration touchpad (Fig. 1) and a tablet (Nexus 9, Google). The touch area on the pad was $13.56 \times 18.08$ cm. An application running on the tablet sent commands for electrovibration rendering to the touchpad. The touchpad and the tablet communicated via an USB On-The-Go cable.

For rendering, we used TanvasTouch SDK. This library represents electrovibration stimuli using grayscale images. Each pixel value indicates the intensity of electrovibration at the corresponding position on the touchpad; 0 (black, no stimulation) to 255 (white, maximum intensity). The physical intensity of frictional vibration is controlled to vary linearly to the command. For this reason, we were not allowed to control the temporal spectral content of electrovibration. This hardware design oriented to spatial pattern rendering was in good agreement with our research goal.

We used Android Studio to implement an Android application for the TanvasTouch device. An experimental program for participants was implemented on a PC using Unity3D. The two programs used Bluetooth to communicate.

### B. Texture Representation

Electrovibration devices are especially suited to rendering textures. Textures can be deterministic or stochastic, and we studied deterministic one in this work. To design deterministic textures in systematic manner, we chose to use regular tessellations. A regular tessellation (tiling) is a spatial pattern made by repeating a regular polygon so that there are no overlaps or gaps [22]. We can obtain diverse spatial patterns by changing several parameters of regular tessellation.

We varied the following five variables to make grayscale images using regular tessellation.

*1) Tile:* Regular tessellation uses a regular polygon, such as an equilateral triangles, square, or regular hexagon, as a tile. We selected two simplest types of tile, an equilateral *triangle* and a *square*, for our experiment.

*2) Density:* We controlled the density of tiles by changing the size of the regular polygon. *Dense* textures used small polygons enclosed by rectangles with a side length of 9 pixels (0.81 mm). The side length for *sparse* textures was 25 pixels (2.21 mm; approximately 2.8 times larger).

*3) Edge Width:* The tiles have borders, and the width of their edges can also affect texture perception. We used two types of edges: *thin* (1 pixel) and *thick* (5 pixels).

*4) Pixel Intensity:* We selected two intensity values for the pixels on the edges: 255 (*strong*) and 127 (*mild*). They represent electrovibration with the maximum frictional force and one with a half of it, respectively.

*5) Image Reversal:* In *normal* images, the edges are colored (white or gray), and the background is black (zero intensity). In *reversed* images, they are switched.

All of the above parameter values were selected after many pilot experiments to provide clear perceptual differences. As a result, we generated 32 texture images that used regular tessellation (two values for each of the five parameters) and used them in the experiment for perceptual dissimilarity judgment. The size of all images were $600 \times 600$ pixels ($54.0 \times 54.0$ mm). The texture parameters are summarized in Table I, which can be compared with Fig. 2.

## III. Methods

We conducted a perceptual experiment to measure the perceptual differences between the 32 regular, homogeneous textures shown in Fig. 2, all rendered by electrovibration. Using these results, we estimated perceptual spaces of the textures. Participants also evaluated the quality of each texture using a number of adjective pairs. This experiment was approved by the Institutional Review Board at the authors' institution (PIRB-2018-E098).

TABLE I

PARAMETER VALUES OF THE 32 TEXTURES USED IN THE PERCEPTUAL EXPERIMENT. S: STRONG; M: MILD, R: REVERSED; N: NORMAL.

| # | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Tile | Square | | | | | | | | | | | | | | | | Triangle | | | | | | | | | | | | | | | |
| Density | Dense | | | | | | | | Sparse | | | | | | | | Dense | | | | | | | | Sparse | | | | | | | |
| Edge width | Thin | | | | Thick | | | | Thin | | | | Thick | | | | Thin | | | | Thick | | | | Thin | | | | Thick | | | |
| Pixel intensity | S | | M | | S | | M | | S | | M | | S | | M | | S | | M | | S | | M | | S | | M | | S | | M | |
| Image reversal | R | N | R | N | R | N | R | N | R | N | R | N | R | N | R | N | R | N | R | N | R | N | R | N | R | N | R | N | R | N | R | N |

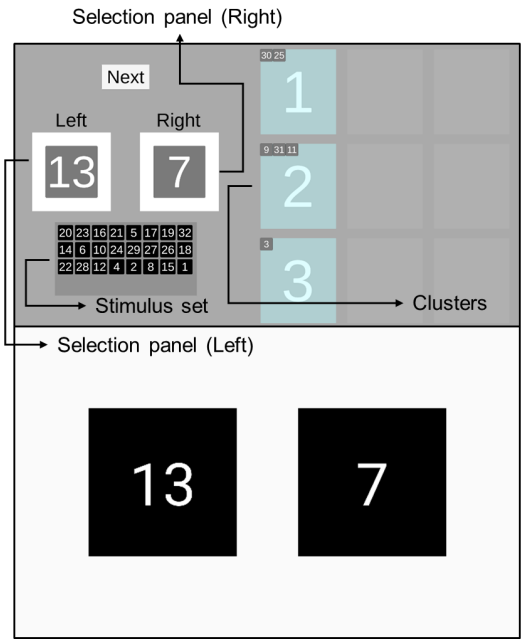



Fig. 3. Top: GUI presented to participants for dissimilarity rating. Bottom: Haptic textures rendered on the Tanvas touchpad. Black rectangles only illustrate the locations of haptic textures.

### A. Participants

Twelve volunteers (six male and six female; 18–25 years old, M: 22.1, SD: 2.2) with no known somatosensory disorders participated in this experiment. They reported that they had not experienced electrovibration prior to this experiment. Participants were paid KRW 15,000 ($\simeq$ USD 13).

### B. Procedure

The experiment consisted of two parts. The first part was for dissimilarity rating between the 32 textures using cluster sorting. Fig. 3 shows our GUI program in the top panel. It initially displayed 32 numbered markers, each representing a texture, at the bottom left. Participants could drag and drop a marker into a left or right selection panel using a mouse. Then they could feel the corresponding texture at the corresponding location on the touchpad. The program rendered two textures side-by-side for participants' easy comparison. The rectangles on the right sides of the screen were for participants to cluster the markers that represented perceptually similar textures.

Participants began with training. They learned how to use the cluster sorting program and experienced all the 32 textures by moving markers into the selection panels one by one. Then they completed three clustering sessions for 3, 6, and 9 clusters. Since the cluster numbers may have significant effects on the results, we followed the guidelines in [23] while maintaining reasonable total experiment time. The orders of conducting the three sessions were balanced using the full factorial ($3! = 6$ permutations) across the 12 participants. Participants' task was to feel and compare all textures pairwise, and then sort them into the specified number of clusters based on similarity. Each cluster was required to include at least one texture. The mapping from marker number to texture was randomly made in each session to prevent participants from relying on memory. Participants could feel all textures freely by moving the markers. They had a rest for 3–5 min between the sessions.

In part 2, participants rated how well the feel of each texture matched to the eight pairs of adjectives with opposite meanings shown in Table II. Our experiment program showed both Korean and English texts to convey more exact meanings (Fig. 4). For each participant, the eight adjective pairs appeared on the screen in such a way that the adjectives in four pairs were switched and the order of all the pairs was randomized. In each trial, participants felt one texture and moved the slider bar (127 mm on the screen) for each adjective pair to rate the extent of agreement of the perceived texture to the adjective pair. All slider bars were initially positioned at the middle. The score (-50–50) for the current slider bar position was shown next to the bar. After evaluating the texture against all the adjective pairs, participants clicked the 'next' button to proceed to the next texture. The 32 textures were presented in random order to each subject.

During the experiment, participants sat in front of the computer and the TanvasTouch, wearing noise-cancelling headphones to block auditory noise. The experiment took approximately one and a half hour per participant.

### C. Data Analysis

The data collected from cluster sorting were processed following the procedure described in [23]. For each participant, the similarity score $s_{i,j}$ between texture $i$ and $j$ was initialized to 0. If texture $i$ and $j$ were grouped together in a session with $N$ clusters, $s_{i,j}$ was added with $N$. For example, if texture 2 and 7 were in the same groups in two sessions with $N = 3$ and 6 (but not 9), its similarity score

TABLE II

EIGHT ADJECTIVES PAIRS USED IN THE RATING EXPERIMENT.

| # | Adjective 1 | Adjective 2 |
|---|---|---|
| 1 | Sharp | Blunt |
| 2 | Dense | Sparse |
| 3 | Vague | Distinct |
| 4 | Jagged | Aligned |
| 5 | Sticky | Slippery |
| 6 | Heavy | Light |
| 7 | Bumpy | Even |
| 8 | Rough | Smooth |

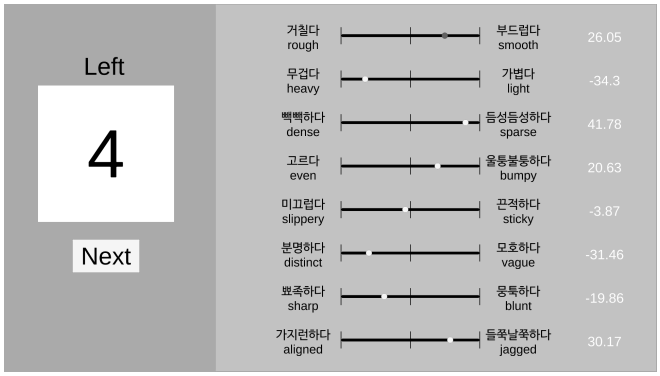



Fig. 4. Screenshot of the experiment program used for adjective rating.

$s_{2,7}$ was $9(=3+6)$. These individual scores were averaged across the participants. Then the average similarity scores were converted to a dissimilarity matrix $\{d_{i,j}|1 \leq i,j \leq 32\}$ by a linear transformation

$$d_{i,j} = 1000\left(1 - \frac{s_{i,j}}{3+6+9}\right),$$

where each element $d_{i,j}$ ranged between 0 and 1000. We then applied non-metric classical MDS to the dissimilarity matrix to find Euclidean perceptual spaces. We used S-Stress ($SS$) to evaluate the goodness of fit [24]. $SS$ takes a value between 0 and 1, and $SS$ closer to 0 indicates better fit.

The results of adjective rating were processed as follows. The score of each adjective pair was linearly converted to 0 for adjective 1 and 100 for adjective 2. These individual scores were averaged across the participants. For each adjective pair, we applied multiple linear regression with the texture positions recovered from a perceptual space as independent variables and the adjective score as a dependent variable. We projected each adjective pair to a line in the perceptual space, which passes the origin and points to the direction proportional to standardized regression coefficients.

## IV. Results And Discussion

### A. Perceptual Space

MDS applied to the average dissimilarity matrix showed that a perceptual space of three dimensions was the best candidate to account for the distribution of texture points, with $SS = 0.1344$ ($SS$ lower than 0.15 represents a good fit [25]). Therefore, we visualized the texture points in a 3D space, as shown in Fig. 5. We also applied hierarchical clustering for non-metric data using the farthest neighbor metric [26].

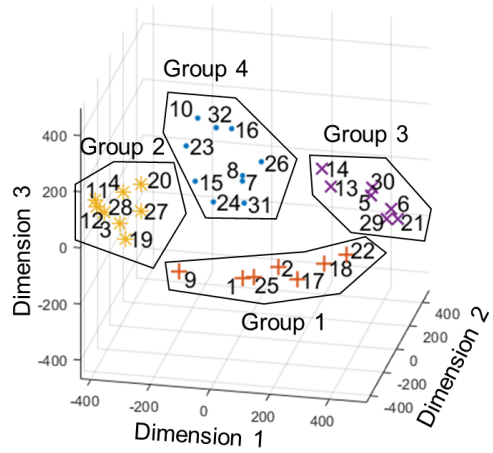



Fig. 5. 3D perceptual space of the 32 textures. Texture groups obtained by hierarchical clustering (Fig. 6) are represented by solid lines.

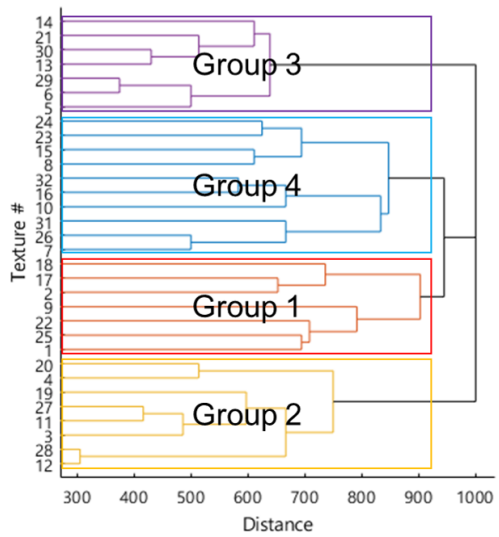
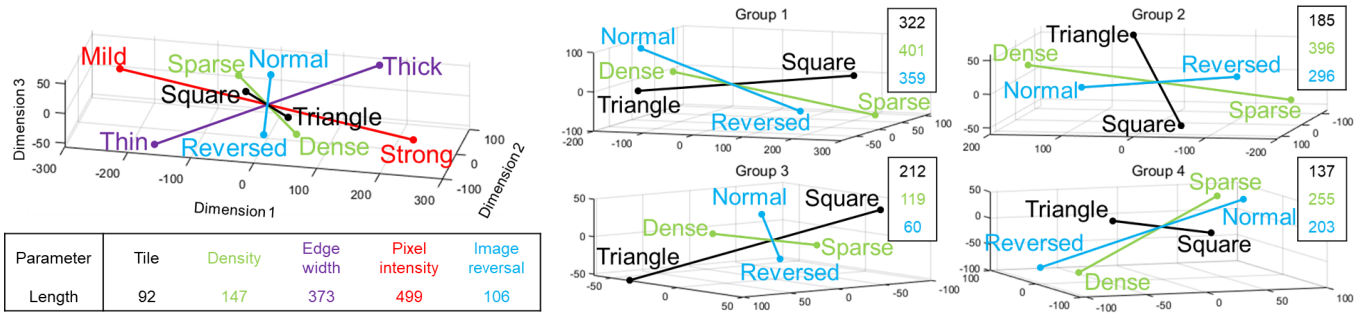



Fig. 6. Dendrogram that shows hierarchical clustering results (Cophenetic correlation coefficient 0.7905).

Results shown in Fig. 6 suggest that the textures can be classified into four groups based on the pairwise perceptual dissimilarity. These four groups are also depicted in Fig. 5, which were also consistent with our visual inspection results.

Group 1 included seven textures {1, 2, 9, 17, 18, 22, 25}, and they had all thin edges and strong pixel intensity, except texture 22. Group 2 had eight textures {3, 4, 11, 12, 19, 20, 27, 28}, all with thin edges and mild intensity. Group 3 had seven textures {5, 6, 13, 14, 21, 29, 30}, and they were composed of thick edges and strong intensity. Lastly, group 4 had 10 textures {7, 8, 10, 15, 16, 23, 24, 26, 31, 32}, all with thick edges and mild intensity except texture 10 and 26. Therefore, edge width and pixel intensity out of the five design variables were dominant in determining the perceptual identities of regular tessellation textures.

For each value of each parameter, our experiment included 16 textures, and their center of mass in the perceptual space is shown in Fig. 7a. Edge width and pixel intensity had the




| Parameter | Tile | Density | Edge width | Pixel intensity | Image reversal |
|---|---|---|---|---|---|
| Length | 92 | 147 | 373 | 499 | 106 |

(a) (b)

Fig. 7. Centers of mass for each value of each texture parameter in the 3D perceptual space (a). The two centers of mass for each parameter are connected by a line for easy recognition. Similar results are shown in (b) for the sub-perceptual spaces of the four texture groups.

widest spread (373 and 499) between their centers of mass. This analysis also suggests that the two design variables had the greatest impact on the texture discriminability.

To obtain further insights, we applied MDS to the sub-confusion matrix of each of the four texture groups, as was done in [23]. We then looked at the centers of masses for each value of the other three design variables, tile, density, and image reversal. Results are shown in Fig. 7b. In group 1, 2, and 4, changing density between dense and sparse resulted in the greatest difference in the perceptual dissimilarity, and switching tile caused the least difference. However, in group 3 composed of the textures with thick edges and strong pixel intensity, the largest dissimilarity change was observed when tile was chanced between triangle and square. It seems that tile makes greater influence than density when the texture is very strong (thick edges and strong intensity). Otherwise, density is more important.

### B. Adjective Rating

The eight adjective pairs in Table II were mapped into the 3D perceptual space of the 32 textures by multiple linear regression, and results are shown in Fig. 8. The $R^2$ values of regression were 0.17, 0.57, 0.77, 0.58, 0.68, 0.76, 0.71, and 0.85, respectively. We also computed the intersecting angles between all pairs of the regressed adjective lines (Table III).

We used two criterions to choose appropriate perceptual dimensions. First, an adjective pair with large $R^2$ is preferred. The perceptual quality of textures varies to a large extent with respect to that pair. Second, the best are mutually orthogonal adjective pairs, wherein the percept can change along one adjective pair completely independently from the others. We searched for those closet to mutual orthogonality by looking at the intersecting angles. Additionally, we considered the general intuitiveness of adjectival axis in describing tactile and texture sensations.

We began with the three prominent perceptual dimensions, i.e., hard-soft, rough-smooth, and sticky-slippery, for bare-finger perception of real textures [13], [18]. Hard-soft was ineligible because electrovibration displays cannot produce stiffness-related physical stimuli. Rough-smooth was still the best one. It had the largest $R^2$ (0.85) and is the most natural and frequently used perceptual dimension for haptic texture.

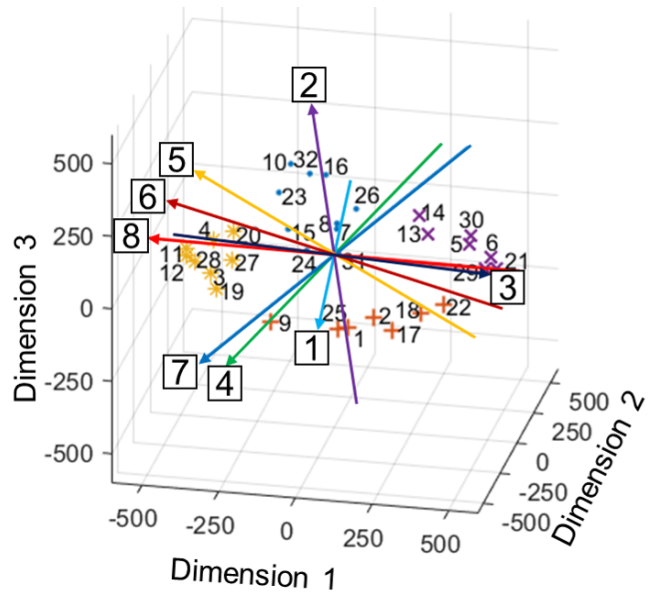



Fig. 8. Multiple linear regression results of the 8 adjective pairs into the 3D perceptual space of the 32 textures. The numbers in squares represent the numbers of adjective pairs in Table II. The length of each line is proportional to $R^2$ of regression. An arrow at one end of each line directs to adjective 2 of the corresponding pair.

TABLE III

INTERSECTING ANGLES IN DEGREES BETWEEN ADJECTIVAL AXES.

| | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 1 | 44.2 | 74.9 | 46.8 | 81.3 | 89.7 | 47.0 | 80.2 |
| 2 | | 70.3 | 51.7 | 49.0 | 62.2 | 60.2 | 75.1 |
| 3 | | | 62.0 | 23.8 | 17.9 | 52.2 | 17.9 |
| 4 | | | | 79.9 | 66.3 | 10.0 | 53.4 |
| 5 | | | | | 13.8 | 70.9 | 26.5 |
| 6 | | | | | | 57.6 | 12.9 |
| 7 | | | | | | | 44.7 |

Many other adjective-pairs exhibited high correlations to rough-smooth, as can be seen in Fig. 8 (line 8 is for rough-smooth). We then selected dense-sparse. It was the most orthogonal one to the rough-smooth axis with a between-axes angle of 75.1° (pair 2 and 8 in Table III). Its $R^2$ was also relatively large (0.57). Dense-sparse also has a natural implication to the distribution of spatial features.

As the third perceptual dimension, our data indicated that sticky-slippery is adequate ($R^2 = 0.68$; its angles from rough-smooth and dense-sparse were 26.5° and 49.0°, respectively). However, we found two even better candidates: bumpy-even (number 7 in Table III; $R^2 = 0.71$; 44.7° and 60.2°[1]) and jagged-aligned (number 4 in Table III; $R^2 = 0.58$; 53.4° and 51.7°). Since these two axes are highly correlated with an intersecting angle of 10.0°, either one can be used. For intuitive interpretation, bumpy-even appears to be rather a better choice (it also has higher $R^2$). Therefore, we conclude that three perceptual dimensions that are most adequate to describe the perceptual attributes of the homogeneous regular textures rendered using electrovibration are rough-smooth, dense-sparse, and bumpy-even.

As a last note, referring to the data in Fig. 8 and Table III carefully can help when the other adjective pairs are used for some other specific purposes.

## V. Conclusions

This paper was concerned with the perceptual characteristics of haptic textures rendered using electrovibration. We modeled homogeneous and deterministic image-based textures using regular tessellation by varying five design variables of tiling polygon, density, edge width, pixel intensity, and image reversal. The textures were evaluated via a perceptual experiment in terms of perceptual pairwise dissimilarity and also compared against a set of adjective pairs. As a result, a 3D perceptual space was estimated from the dissimilarity data. Its results were supplemented with the adjective pairs projected into the perceptual space to account for the space structure with linguistically meaningful axes.

The major findings can be summarized as follows: 1) The percepts of regular and homogeneous image-based textures rendered by electrovibration can be well represented in a 3D perceptual space; 2) The perceptual identity of a texture is primarily determined by its edge width and pixel intensity; 3) When the texture sensation is sufficiently strong, people may be able to distinguish the polygonal shapes of tiles and then it is of the next importance for identity; 4) If not, the density of repeated features is of the next significance; and 5) Adjective pairs recommended as three perceptual dimensions of the perceptual space are rough-smooth, dense-sparse, and bumpy-even. We expect that these guidelines facilitate the design of perceptually distinct textures with anticipated perceptual attributes displayed on an electrovibration display.

In this study, we used only two values for each parameter. Using more values prolonged the experiment duration to an unbearable level for participants. Given the notable effects of edge width and pixel intensity, follow-up studies focusing on the two variables may reveal additional insights on the perceptual nature of textures induced by electrovibration.

## Acknowledgements

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[1]The similar and larger these two angles are, the closer the three lines are to the mutual orthogonality.

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